

WOODS HOLE OCEANOGRAPHIC INSTITUTION

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The 1961 Summer Program of Theoretical Studies in  
Geophysical Fluid Dynamics

by

George Veronis

Final Report

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APPROVED FOR DISTRIBUTION \_\_\_\_\_

Paul M. Fye, Director.



Description of the program:

This ten-week work-study-discussion program was centered about a formal course called Geophysical Fluid Dynamics. Sixteen participants were selected from graduate and postgraduate applicants. In the discussions emphasis was placed on the formulation of tractable research problems in geophysics. The participants were encouraged to work on satisfactory problems thus formulated and to continue with their research after returning to their respective institutions.

Participants supported by the National Science Foundation grant:

Alewyn Burger, Ph.D., Mathematics, Delft University, Holland  
Director Institute of Mathematical Sciences,  
Pretoria, South Africa

George B. Field, Ph.D., Astronomy, Princeton University  
Associate Professor Astrophysics, Princeton University

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Robert H. Kraichnan, Ph.D., Physics, MIT  
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Frank B. Lipps, Ph.D., Meteorology, Johns Hopkins University  
Research Associate, Department Geophysical Sciences, University of Chicago

Derek Moore, Ph.D., Mathematics, Cambridge University, England  
Lecturer in Mathematics, University of Bristol, England

Edward Spiegel, Ph.D., Astronomy, University of Michigan  
Research Associate Institute of Mathematical Sciences, New York University

Ivar Dugstad, B.S. University of Oslo, Norway, 1954  
M.S. Texas A&M, 1956. Research Associate at Institute of Theoretical Meteorology, University of Oslo, Norway

- Lionel W. MacMillan, B.A., 1950; M.S., 1957, University of Saskatchewan, Canada. Ph.D. candidate in Physics at University of British Columbia
- Martin T. Mork, B.S., 1958, University of Oslo, Norway. Research Associate at Oceanographic Institute, University of Oslo, Norway
- S. Nagarajan, B.Sc., 1956, Presidency College of Madras University, India. Graduate student at Institute of Mathematical Sciences, New York University, Physics Department
- Pearn P. Niiler, B.Sc., 1960, Lehigh University. Graduate student at Brown University, Department of Applied Mathematics
- Dennis H. Peregrine, B.A., 1960, Jesus College, Oxford University, England. Ph.D. candidate in Fluid Mechanics at Churchill College, Cambridge University, England
- Steven I. Rosencrans, S.B., 1960, Massachusetts Institute of Technology. Graduate student in Mathematics, Massachusetts Institute of Technology
- Fred C. Shure, A.B., 1955, Harvard College, in Physics. M.S., 1957, and graduate student in Physics at University of Michigan

3. Course Topics for 1961:

The four-week formal course was presented by Dr. George Veronis. The purpose was to give examples of two types of research in geophysical fluid dynamics. The course was given at an advanced level and subsumed either a familiarity with basic fluid dynamics or sufficient maturity to obviate the necessity for giving a basic fluid dynamics course. The lectures covered thermal convection and the evolution of theories of wind-driven oceanic circulation.

Dr. N. P. Fofonoff was the invited guest lecturer and gave six lectures on "Energy Transformation in Sea Water". He discussed the transformation of potential energy to kinetic energy through mixing processes in sea water.

The summer program included informal one-hour seminars on various topics. These are listed in the appendix.

4. Student lectures in 1961:

An important part of this program is the requirement that graduate participants prepare a one-hour lecture on original material. Their work can represent an extension of some topic covered in the course or an application of the evolved techniques to some geophysical problem. They are assisted by the staff both in the choice of their topic and as their work progresses. However, it is made clear that the originality of their effort will be the measure of its success.

Ten National Science Foundation fellows and one independently-supported participant spoke at the end of the 1961 program. The speakers and their topics are given in the appendix under "Contents of Volume III".

All of the participants made a considerable effort to prepare a stimulating and original lecture.

5. Publication of the lectures and student research studies:

The notes of the summer course were taken by students assigned to each lecture. The edited version of these notes, the invited lecture series and the students' reports proved to be both a valuable record of the summer effort and a real contribution to geophysical fluid dynamics. They have been reproduced in three volumes and are available to interested persons. The introduction and table of contents are included as an appendix to this report.

6. Plans for the future:

The type of program presented in 1961 is oriented toward the student. For 1962, we plan to submit a proposal which is even less formal and is primarily oriented toward the staff. Since the majority of the staff comes to Woods Hole from universities, its close contact with students of the GFD course represents an intensification of the type of task with which it is faced during the academic year. Thus the plans for 1962 are meant to provide more intense communication between staff members and to minimize the duties associated with the education of students. In the summer of 1963 we propose to return to the type of program given in 1961.

Appendix  
of a final report to the National Science Foundation

Notes on the 1961  
Summer Study Program  
in  
GEOPHYSICAL FLUID DYNAMICS  
at  
The WOODS HOLE OCEANOGRAPHIC INSTITUTION

Contents of the Volumes

Volume I. Student Notes of Lectures by George Veronis  
on Geophysical Fluid Dynamics.

Volume II. Lectures by N. P. Fofonoff  
on Energy Transformations in the Ocean.

Volume III. Participants' Lectures.

## LIST OF PARTICIPANTS

### Regular WHOI Staff Members

C. Rooth  
G. Veronis

### Visiting Staff Members and Post-doctoral Participants

A. Arking (NASA)  
A. P. Burger (NPRL, Pretoria, South Africa)  
G. B. Field (Princeton University)  
N. P. Fofonoff (Nanaimo, British Columbia)  
S. Fritz (U.S. Weather Bureau)  
A. S. Furumoto (University of Hawaii)  
J. Herring (NASA)  
L. N. Howard (MIT)  
R. H. Kraichnan (New York University)  
F. B. Lipps (Johns Hopkins University)  
W. V.R. Malkus (UCLA)  
D. W. Moore (Bristol University, England)  
E. A. Spiegel (New York University)  
H. Stommel (Harvard University)  
P. Welander (Nat'l. Research Council, Stockholm, Sweden)

### Student Fellows

I. Dugstad	Met.-Oceanog.	Univ. of Oslo, Norway
L. W. MacMillan	Physics	Univ. of British Columbia Vancouver, Canada
M. T. Mork	Met.-Oceanog.	Univ. of Oslo, Norway
S. Nagarajan	Physics	New York University
P. P. Niiler	Applied Math.	Brown University
D. H. Peregrine	Applied Math.	Cambridge University, England
S. I. Rosencrans	Mathematics	Mass. Inst. Tech.
F. C. Shure	Physics	University of Michigan



Notes on the 1961  
Summer Study Program  
in  
GEOPHYSICAL FLUID DYNAMICS  
at  
The WOODS HOLE OCEANOGRAPHIC INSTITUTION

Volume I  
Student Notes of  
Lectures by George Veronis  
on  
GEOPHYSICAL FLUID DYNAMICS

First Volume Edited by  
W. V.R. Malkus  
University of California at Los Angeles

### Editor's Preface

One expects an Oceanographic Institution to encourage the study of the motions of the sea. However, an explanation is needed for the Woods Hole tradition of teaching and research across the entire spectrum of geophysical fluid dynamics. The motions in the earth's core and in its liquid and gaseous envelopes, the motions in the sun and in its tenuous atmosphere; all are flows caused by the convection of heat and the transfer of momentum in rotating systems. Understanding in one area is usually a contribution to understanding in all. By encouraging study with a broad base the theoretical group at Woods Hole has been able to gain from the enthusiasms and skills of meteorologists, physicists, mathematicians and astrophysicists.

This report of the third summer course in Geophysical Fluid Dynamics attests to the new tools and ideas, drawn from many disciplines, which now enrich oceanographic study.

The success of this third course is due primarily to the efforts of Dr. George Veronis. He has acted both as Director of the study program and as the principal staff lecturer.

The first volume of these notes is a restatement by the students of Dr. Veronis' introductory lectures. Despite the considerable differences in presentation, the notes preserve the content and spirit of the lecture material.

The editor and students wish to thank Mrs. Mary Thayer for her capable assistance in assembling and reproducing these notes. We are all indebted to the National Science Foundation for its continuing support of the Summer Study Program.

Willem V.R. Malkus

### Lecturer's Foreword

Any attempt to lecture on a field as broad as theoretical geophysical fluid dynamics must reflect the lecturer's philosophy. My own view is that the study of geophysical hydrodynamics is composed of a combination of three types of investigation. In decreasing order of rigor they consist of theoretical investigations into: a) fluid processes of a general nature; b) laboratory experiments modelled on a specific geophysical phenomenon; and c) theoretical models of actual geophysical phenomena.

In the limited time allowed for these lectures I chose to treat an example of the two extremes. As an example of class a) thermal convection is treated in a quite rigorous fashion with the analysis proceeding from the pure conductive state through the stability problem to finite amplitude motions and finally to fully-developed turbulence. In class c) the steady, wind-driven, ocean circulation is the area of investigation. The attempt has been to study the evolution of the theory from the earlier linear models to the latest models which are non-linear.

It was not intended that the students' notes be a simple record of the lectures. In some cases the material was reworked by the student and the presentation reflects much of the student's own orientation and interest. In other cases, the notes are much closer to the presentation in the lecture room. In all cases, the outline of the lectures has been adhered to and the material is essentially that covered in the lectures.

George Veronis

Veronis Lectures

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Volume III

PARTICIPANTS' LECTURES

Third Volume Edited by

Mary C. Thayer and George Veronis



### Editors' Preface

This volume contains the manuscripts of the student research lectures as well as research contributions by senior participants in the summer program. The staff guided the selection of the students' topics with several goals in mind. One goal was to isolate that part of a problem which might prove to be tractable in an effort of eight weeks or so. The more important goal was to find "open-ended" problems which would continue to challenge the student after his return to the university.

The degree of direction by the sponsor varied a great deal. In a few cases, there were frequent conferences and discussions about fruitful avenues of approach. In other cases, there was essentially no contact except one of encouragement and interest. The efforts cover a wide spectrum in originality also. Some of the reports represent a more extended study of material presented in the course of lectures - others are original contributions which are being prepared for publication.

Because of time limitations it was not possible for the notes to be edited and reworked. The reports may contain errors the responsibility for which must rest on the shoulders of the participant-author. It must be emphasized that this volume in no way represents a collection of reports of completed and polished work.

In cases of joint-authorship the names have been listed alphabetically with no attempt to distinguish between senior and junior contributor.

All those who took part in the summer program are grateful to the National Science Foundation for its encouragement and financial support of the program.

Mary Thayer

George Veronis

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Lionel W. MacMillan. "Long" Waves on a Rotating Globe

Derek W. Moore. Two-dimensional Motions in a Homogeneous  
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Martin T. Mork. Circulation in a Rectangular Basin  
derived from a Source and Sink Model

S. Nagarajan. Time Dependent Motions in Thermal Convection

Pearn P. Niiler. Integral Relations in a Variable Density  
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Dennis H. Peregrine. The Effect of Temperature Variations  
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Steven I. Rosencrans. Penetrative Convection

Fred C. Shure. Combined Shear Flow and Convection

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